A simple introduction to Fourier X-ray timing

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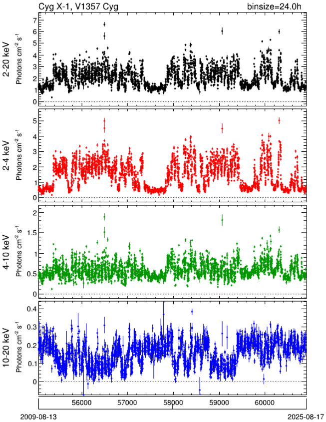
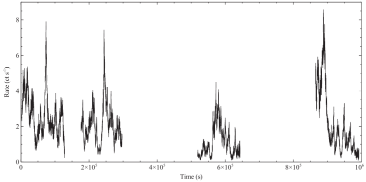
Motivation

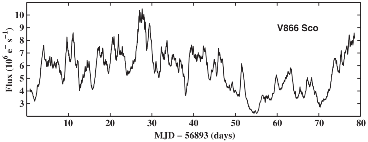
- Many astronomical objects, especially accreting systems, are highly variable systems
- We a mathematical tool to extract timing info



Cyg X-1 (accreting stellar-mass BH; MAXI)



IRAS 13224−3809 (accreting SMBH; Fabian+13)



V866 Sco (accreting protostar; Scaringi+15)

Fourier transform

- Fourier transform:
$$F(\omega) = \int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt.$$
- And the inverse transform:
$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)e^{i\omega t}d\omega.$$
which says that a function of time can be seen as superposition of periodic functions of different variability frequencies

Discrete Fourier Transform

- From astronomical observations, we always obtain discrete time series instead of continuous ones
- Consider an X-ray observation, with N time bins, time resolution dt, counts in the k-th bin x_k
- To apply Fourier analysis, we need to discrete the Fourier transformation (Discrete Fourier Transform, DFT):

$$a_j = \sum x_k e^{2\pi ijk/N},\ j = -N/2, \cdots, N/2,$$
$$\nu_j = j/Ndt : \nu_{\min} = 1/Ndt,\ \nu_{\max} = 1/2dt.$$

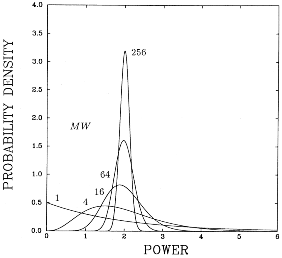
- a_j is complex and contains both the variability amplitude and phase at the frequency ν_j. If we only care about the variability amplitude, we compute the square of a_j to obtain the power spectral density (PSD):

$$P_j = |a_j|^2.$$

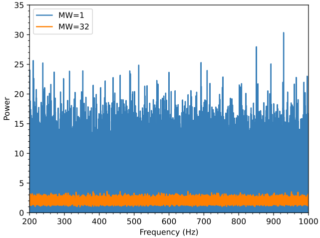
- Two commonly adopted PSD normalization:
  - Leahy normalization: $P_j = \frac{2}{N_{\rm ph}}|a_j|^2$.
  - rms normalization: $P_j = \frac{2Ndt}{N_{\rm ph}^2}|a_j|^2$.

Power statistics

- If x_k follows Poisson statistics and are un-correlated, then under the Leahy normalization, P_j follows the χ^2 distribution with 2 degree of freedoms (dof); $\bar{P}_j = 2$. This is known as the white noise (noise independent of frequency)
- χ^2_2 distribution is very broad and therefore the PSD is noisy. In practice we usually
  - split time series into M segments, and average PSD
  - rebin the PSD to average over W consecutive frequency bins
  - the power follows a χ^2_{2MW} distribution scaled by $1/MW$



χ^2 dist. of various dof; van der Klis 89

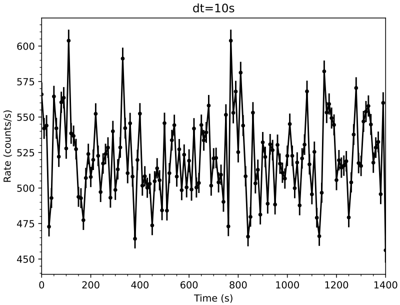


White noise with different values of MW

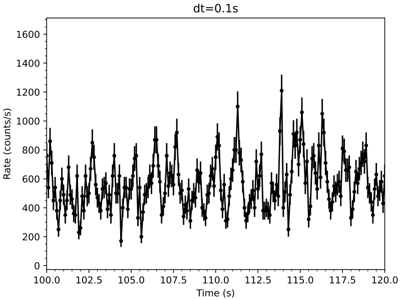
Pulsation search using powspec in a NICER dataset

Dataset

- ni3602020701_0mpu7_cl.evt.gz: the event list of a NICER observation of Her X-1, an accreting X-ray pulsar with a spin period of ∼1.24 s



Lightcurve of Her X-1



Lightcurve of Her X-1, 0.1 s time resolution

Pulsation search using powspec in a NICER dataset (exercise)

Pulsation search

1. Extract source 0.5-10 keV event using *xselect*
2. Perform barycentric correction using *barycorr*

```
barycorr infile=herx1_05-10.evt \
outfile=herx1_05-10_barycorr.evt \
orbitfiles=ni3602020701.orb ra=254.4575 dec=35.3424 \
barytime=yes
```

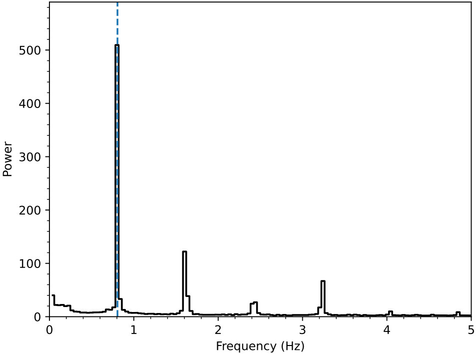
3. Computing PSD using *powspec*

```
powspec cfile1=herx1_05-10_barycorr.evt \
normalization=1 dtnb=4e-4 window=- nbint=65536 \
nintfm=INDEF rebin=0 outfile="" plot=yes plotdev=/xw
```

powspec parameters

- cfile1: input event file, lightcurve file, or a file list
- normalization: switch for normalization. 1(-1) and 2(-2) for Leahy and rms normalization, respectively. If negative, the white noise is subtracted.
- dtnb: the duration of newbin for timing analysis. Should be integer multiple of the intrinsic time resolution.
- nbint: the number of newbin in a segment
- nintfm: the number of segments in a frame
- rebin: positive: linear rebin; negative: logarithm rebin; 0: no rebin

PSD of Her X-1

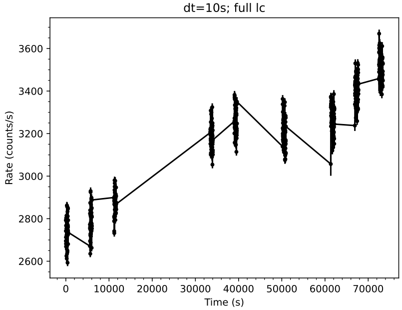


- A sharp peak at 1/1.24 Hz clearly seen (also harmonics), demonstrating the power of Fourier analysis
- Significance of the peak can be estimated knowing the probability of the white noise
- For pulsars in binary, extra time-delay due to binary motion has to be taken into account (see e.g. Ransom S. M. 2001, Ph.D. Thesis)

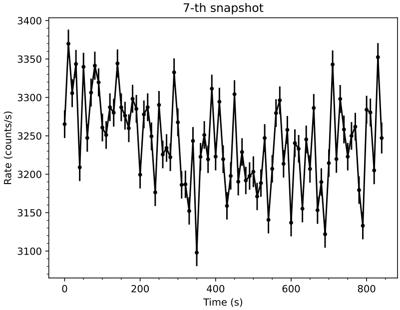
Fourier analysis for aperiodic variability

Dataset

- ni4133010103_0mpu7_cl.evt.gz: the event file of a NICER observation of GX 339-4, an accreting stellar-mass black hole



The full lightcurve; 10-s bin



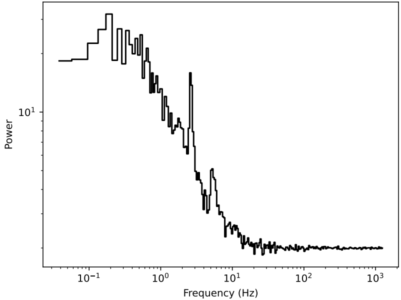
Lightcurve of the 7-th snapshot

Fourier analysis for aperiodic variability (exercise)

Producing PSD with powspec

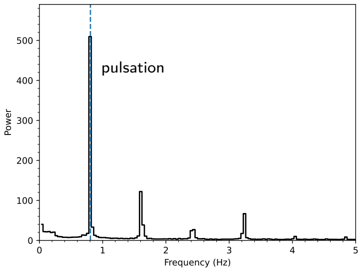
1. Extract the 0.5-10 keV event list of the 7-th snapshot using *xselect*
2. Computing PSD using *powspec*

```
powspec cfile1=gti7.evt normalization=1 dtnb=4e-4 \
window=- nbint=65536 nintfm=INDEF rebin=-1.03 \
outfile=gx339.fps plot=yes plotdev=/xw
```



Features in PSD

- Pulsation (narrow peak)
- Quasi-periodic oscillation (QPO; broad peak)
- Band-limited noise (BLN; constant at low frequencies, rapid decay above a critical frequency)
- White noise (constant power with respect to frequency)
- Red noise (“red” since higher power towards lower frequencies)



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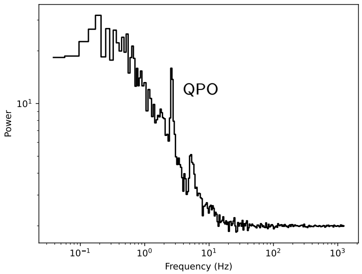

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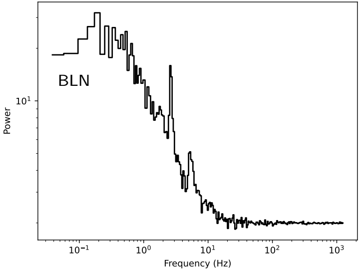

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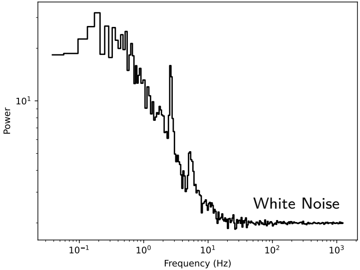

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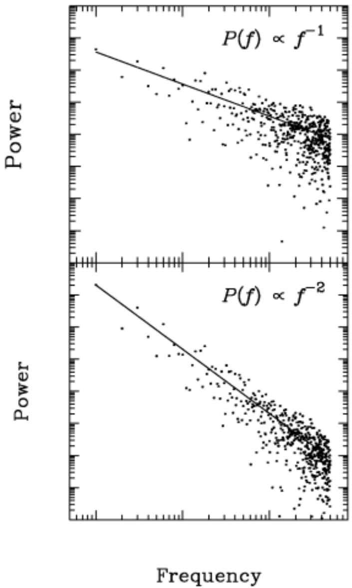
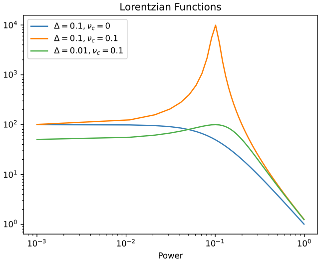


PSD fitting

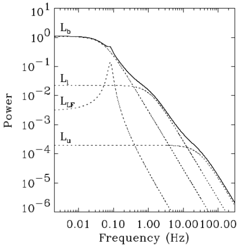
- Having produced the PSD, we need to measure the parameters of different components
- In the X-ray binary community, the most popular way to fit the PSD is using the Lorentzian function to fit BLN and QPO in a unified fashion

$$L(\nu) \propto \frac{1}{\Delta^2 + (\nu - \nu_c)^2}.$$

- In practice, the PSD can be fit with Xspec, Stingray, or any fitting tool you like



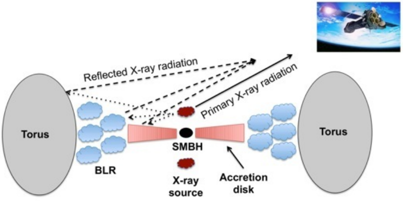
Lorentian functions



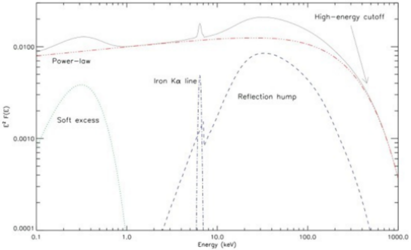
Lorentian fit of a PSD; Belloni+02

Fourier time-lag analysis

- Time-lag between different spectral component contains valuable info
- For instance, the time-lag between hard X-ray emission and the reflection component helps us constrain the coronal geometry



Cartoon of AGN disc/corona. Credit: R. Ricci



AGN X-ray spectral components. Credit: R. Ricci

Fourier time-lag analysis

- Frequency-dependent time-lags between two time series $f_1(t)$ and $f_2(t)$ can be computed by:

$$\tau(\omega) = \frac{\arg(F_1(\omega)F_2^*(\omega))}{\omega},$$

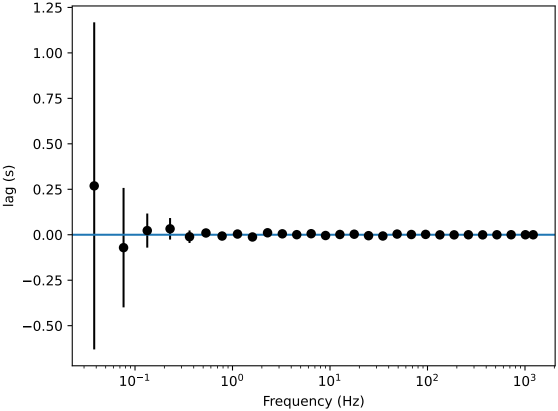
where $F_1(\omega)$ and $F_2(\omega)$ are the DFT of f_1 and f_2, respectively, arg means argument, and $*$ denotes complex conjugation.

- Computing time-lags with *Stingray*:

```
from stingray.fourier import avg_cs_from_events
from stingray import AveragedCrossspectrum, EventList
# Note that event_ref is the reference band
event_ref = EventList.read("gti7_2-4.evt", "hea")
event_sub = EventList.read("gti7_4-10.evt", "hea")
cs = AveragedCrossspectrum.from_events(event_sub,
    event_ref, segment_size=26.2144, dt=4e-4)
cs_reb = cs.rebin_log(0.4)
lag, lag_e = cs_reb.time_lag()
```

Fourier time-lag analysis

- We compute the 4-10 keV vs. 2-4 keV time lag
- No significant lag is detected in this case



4-10 keV vs. 2-4 keV time lag, for the 7-th snapshot of the GX 339-4 dataset

References

- van der Klis, M. 1989, Fourier techniques in X-ray timing, `https://ui.adsabs.harvard.edu/abs/1989ASIC..262...27V/abstract`
- Bachetti, M. & Huppenkothen, D. 2022, Fourier Domain, arXiv:2209.07954, `https://ui.adsabs.harvard.edu/abs/2022arXiv220907954B/abstract`